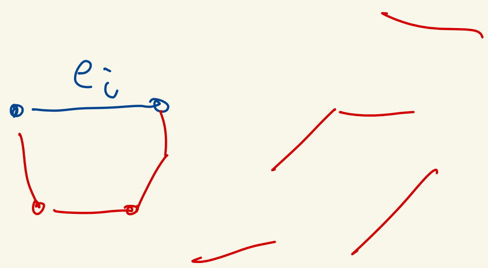
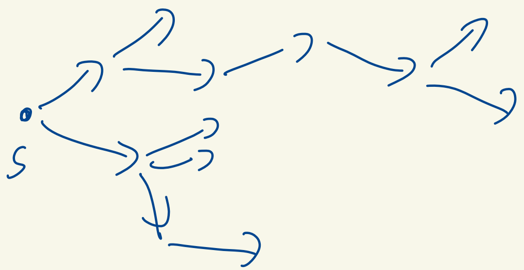
Easy: Minimum spanning tree

1. Kruskal's greedy algorithm

$$w(e_1) \le w(e_2) \le \ldots w(e_i) \le w(e_m)$$



2. out-branchings from s in $D=(V,A)$

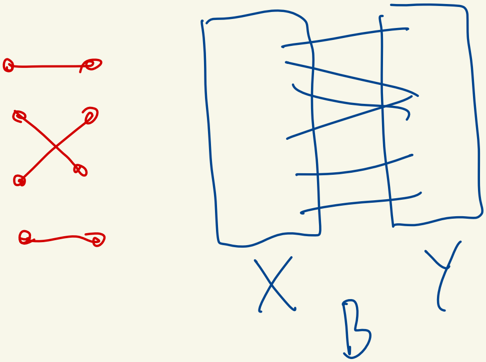


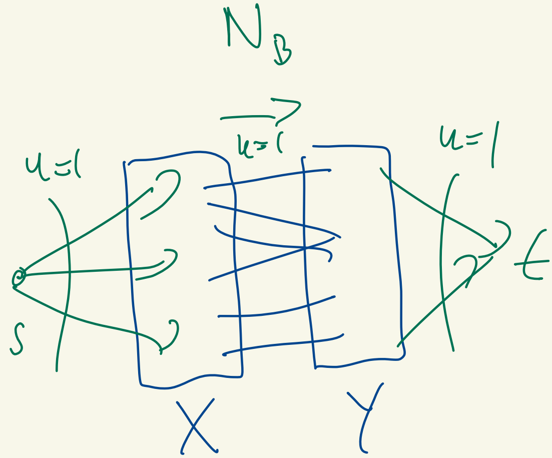
M_1 forest in $UG(D)$

M_2 indegree ≤ 1 and 0 for s

Check by doing a Breath-first/Depth-... from s

3. Bipartite matching



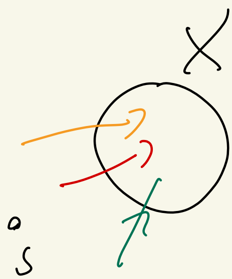


value of max (s,t)-flow in N_B

= size of maximum matching in B

4. Given $D=(V,A)$ $s \in V$ and integer k

Does D have k-arc-disjoint out-branchings from s?

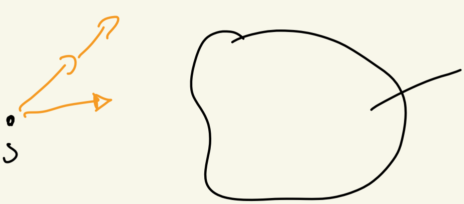


Thm (Edmonds)

The out-branchings exist

$\Leftrightarrow$

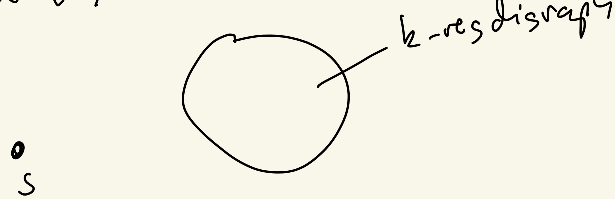
$$\forall X \subseteq V-s \quad d^-(X) \geq k$$



not orange yet

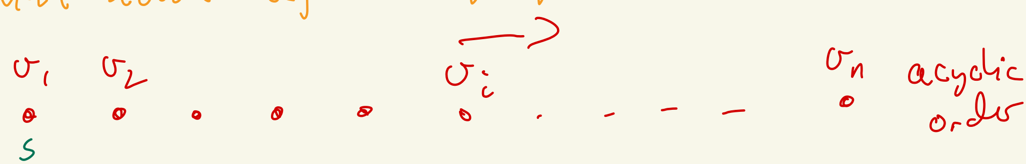
not enough to take $k(n-1)$ arcs so that

$d^-(v) = k \quad \forall v \in V-s$



5. NPC (Zirui?) to decide whether $D=(V,A)$ has an out-branching B_s^+ s.t. $D - A(B_s^+)$ is connected (in the underlying sense)

what about acyclic digraphs?



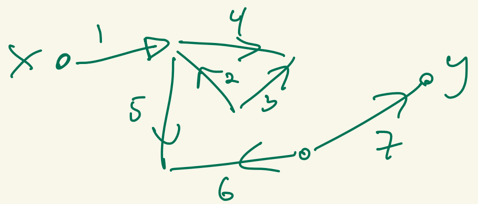
Easy part: D has an out-branching from s

$\Updownarrow$

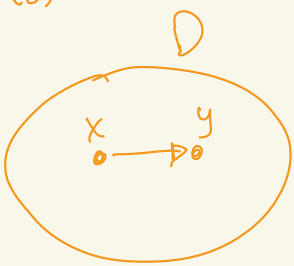
$\forall v_i$ there is at least one v_j $j<i$ s.t. $v_j \to v_i$ ($\square$)

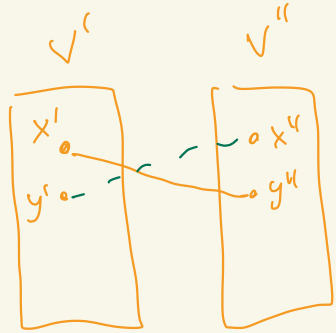
<u>Conclusion:</u> we need to find a spanning tree T in $UG(D)$ s.t $(\square)$ holds in $D - A(T)$

6. $D=(V,A)$ is <u>antistrong</u> if $\forall x \neq y$ $x,y \in V$ then exist a (x,y)-antidirected trail

s.t 

D is antistrong if and only if $B(D)$ is connected





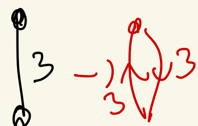
How do we decide whether a given <u>graph</u> can be oriented as an antistrong digraph?

7. Given $D=(V,A)$, $s\in V$ $\quad w: A\to\mathbb{Z}^+$
k integer

s.t D has k arc-disjoint out branchings from s.

Find arc-disjoint $B^+_{s,1} \dots B^+_{s,k}$ s.t total weight of them is minimized

(even $k=1$ is interesting)



(similar find in G undirected k edge-disjoint spanning trees of minimum total weight)

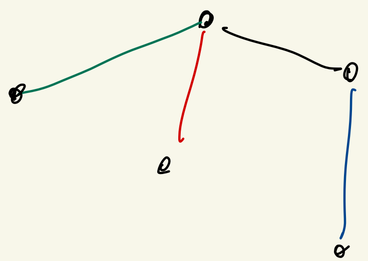
8. suppose G is the union of two spanning trees T_1, T_2 and $w: E\to\mathbb{Z}^+$

s.t $w(T_1)+w(T_2)$ is minimum 10010

Can we balance them?

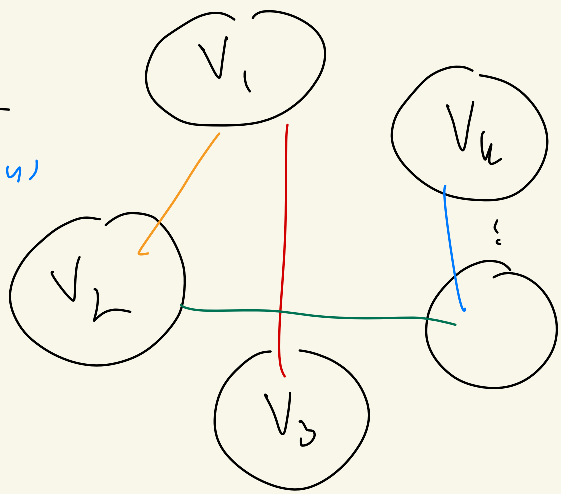
NPC

9. Rainbow spanning trees in edge coloored graphs.



How to decide if such a tree exists in G?

Necessary:



for all partitions $V_1, V_2 \dots V_k$ and all $k \geq 2$, $k \leq n$

at least $k-1$ different colours

Suzuki-Schrijver